OCR
Oxford Cambridge and RSA

Wednesday 17 May 2017 – Morning

A2 GCE MATHEMATICS

4729/01 Mechanics 2

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4729/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Answer **all** the questions.

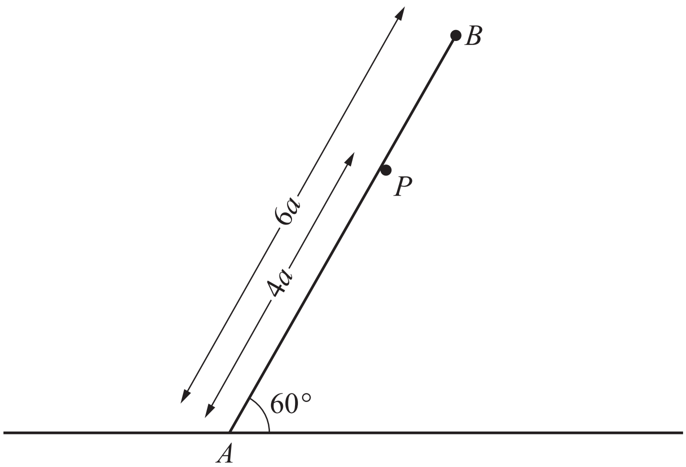
1 A boy drags a sledge in a straight line along horizontal ground by means of a rope attached to the sledge. The rope makes an angle of 25° with the horizontal and the tension in the rope, T N, is constant. The work done by the tension in moving the sledge 75 m is 4000 J. Calculate the value of T. **[3]**

2 A golfer hits a ball from a point O on horizontal ground with a velocity of $55\,\mathrm{m\,s^{-1}}$ at an angle of 20° above the horizontal. The ball first hits the ground at a point A and the time of flight is t seconds. Assuming that there is no air resistance, calculate

(i) the value of t and the distance OA, **[4]**

(ii) the speed and direction of motion of the ball 2.6 s after the golfer hits the ball. **[5]**

3



A uniform rod AB of mass m and length $6a$ rests in a vertical plane with A on rough horizontal ground. A particle of mass km, where k is a constant, is attached to the rod at B. The rod makes an angle of 60° with the horizontal and is supported by a small smooth peg P. The distance AP is $4a$ (see diagram).

(i) Calculate, in terms of m, g and k, the magnitude of the force exerted by the peg on the rod. **[4]**

The coefficient of friction between the rod and the ground is $\frac{1}{3}\sqrt{3}$.

(ii) Find the greatest value of k for which the rod remains in equilibrium. **[5]**

4 A car of mass 1200 kg travels up a line of greatest slope of a straight road inclined at 4° to the horizontal. The power of the car's engine is constant and equal to 23 kW and the resistance to the motion of the car is constant and equal to 800 N. The car passes through a point A on the road with speed $8\,\text{m s}^{-1}$.

(i) Find

(a) the acceleration of the car at A,

(b) the greatest steady speed at which the car can travel up the hill. **[5]**

The car later passes through a point B on the same road where $AB = 109$ m and the car takes 10.1 s to travel from A to B.

(ii) Calculate the speed of the car at B. **[7]**

5

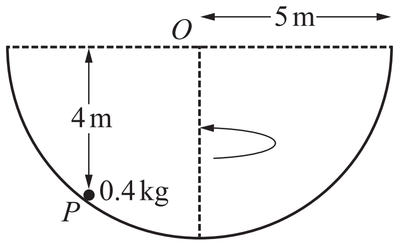


Fig. 1

A particle P of mass 0.4 kg is moving on the smooth inner surface of a fixed hollow hemisphere which has centre O and radius 5 m. P moves with constant angular speed in a horizontal circle at a vertical distance of 4 m below the level of O (see Fig. 1).

(i) Find, in terms of g, the magnitude of the force exerted by the hemisphere on P. **[3]**

(ii) Show that the time taken, in seconds, for P to complete one revolution of its circular path is given by $\frac{4\pi}{\sqrt{g}}$. **[4]**

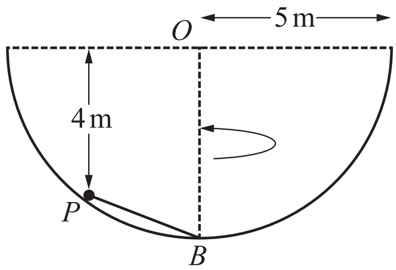
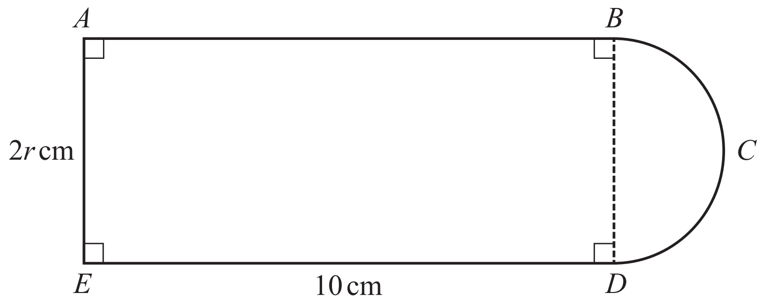


Fig. 2

One end of a light string is now attached to P. The other end of the string is attached to the lowest point B of the hemisphere. P moves in the same horizontal circle as before (see Fig. 2).

(iii) Given that the tension in the string is $\sqrt{40}$ N, calculate the new speed of P. **[6]**

6



A uniform lamina *ABCDE* consists of a rectangle *ABDE* and a semicircle *BCD* joined along the common edge *BD*. $AB = DE = 10$ cm and $AE = BD = 2r$ cm (see diagram).

(i) Show that the distance of the centre of mass of the lamina from *AE* is

$$\frac{4r^2 + 30\pi r + 600}{3(40 + \pi r)} \text{ cm.}$$ **[5]**

The lamina is freely suspended from *B* and hangs in equilibrium. The angle between *AB* and the downward vertical at *B* is θ, where $\tan\theta = \frac{1}{4}$.

(ii) Show that *r* satisfies the equation

$$(3\pi + 1)r^2 + 120r - 150 = 0,$$

and hence find *r*. **[5]**

7 Two small spheres *A* and *B*, of masses 4 kg and 2 kg respectively, are moving in opposite directions along the same straight line towards each other on a smooth horizontal surface. *A* has speed $1\,\text{m}\,\text{s}^{-1}$ and *B* has speed $3\,\text{m}\,\text{s}^{-1}$ before they collide. The coefficient of restitution between *A* and *B* is *e*. The directions of motion of both *A* and *B* are reversed as a result of the collision.

(i) Find, in terms of *e*, the speed of each sphere after the collision and hence show that $e > \frac{1}{8}$. **[7]**

The total loss in kinetic energy due to the collision is 2.5 J.

(ii) Show that $e = \frac{7}{8}$. **[4]**

A third small sphere *C* of mass 3 kg is moving in the same straight line as *A* and *B*. After the collision between *A* and *B*, sphere *B* subsequently collides with *C*. The coefficient of restitution between *B* and *C* is *f*, and immediately before this collision *C* is moving with speed $3\,\text{m}\,\text{s}^{-1}$ in the opposite direction to *B*.

(iii) The direction of motion of *C* is unchanged by the collision between *B* and *C*, and subsequently *B* collides with *A* again. Find the set of possible values of *f*. **[5]**

Question			Answer	Marks	Guidance
1			$T\cos 25$ $4000 = (T\cos 25)(75)$ $T = 58.8$ N	B1 M1 A1 **[3]**	Seen in a WD equation WD = $F \times d$, must have component of T 58.8468…
2	(i)		$0 = (55\sin 20)t - \frac{1}{2}(9.8)t^2$ $t = 3.84$ s $OA = (55\cos 20)t$ = 198 m	M1 A1 M1 A1 **[4]**	Use of $s = ut + \frac{1}{2}at^2$ vert. with $s = 0$ or use of $v = u + at$ with $v = 0$ and double t found or $T = (2u\sin\theta)/g$ or use symmetry and $v = u + at$ or use OA found from equation of trajectory 3.83900… Use of $s = ut$ horizontally with cv(t) or $R = (u^2\sin 2\theta)/g$ or use equation of trajectory with $y = 0$ 198.4114…
2	(ii)		$v_x = 55\cos 20$ $v_y = 55\sin 20 - 9.8(2.6)$ $v = \sqrt{v_x^{\ 2} + v_y^{\ 2}}$ or $\tan\theta = \frac{v_y}{v_x}$ $v = 52.1\text{ m s}^{-1}$ $\theta = 7.35°$ below horizontal	B1 B1 M1 A1 A1 **[5]**	51.68309… ±6.668892… Use of Pythagoras or relevant trig on cv(v_x) and cv(v_y) 52.11157… AEF; 7.352494…; direction may be shown on diagram with minimum of arrow on resultant or arrows on both components
3	(i)		 $3a(mg\cos 60) + 6a(kmg\cos 60) = 4aR$ $R = \frac{3}{8}mg(1 + 2k)$	M1 A1 A1 A1 **[4]**	Moments about A (oe) – 3 terms but 4 terms if components of R used. Moments about other points needs a complete method. Allow with omission of g. A1 for two terms correct AEEF

Question			Answer	Marks	Guidance
3	(ii)		$X = R\sin 60$	B1	Resolving horizontally
				M1	Resolving vertically, 4 terms, component of R Award if taking moments (all relevant forces included) about any point (not A). If Horizontal resolution is replaced this way, give M1A1 to either the vertical or a moments equation and B1 to the other. Similarly if 2 moments equations.
			$Y + R\cos 60 = kmg + mg$	A1	
			[$\frac{3\sqrt{3}}{16}mg(1+2k) = \frac{1}{16}mg\mu(13+10k)$]	M1	Use of $X = \mu Y$ with cv(R) and μ substituted and $Y \neq R$ from (i)
			$k_{max} = \frac{1}{2}$	A1 **[5]**	Allow $k \leq \frac{1}{2}$
4	(i)		Driving force $= \frac{23000}{v}$ or $\frac{23000}{8}$	B1	
			$1200g\sin 4$	B1	
			$\frac{23000}{v} - 800 - 1200g\sin 4 = 1200a$	M1	N2L with either 3 or 4 terms
			$v = 8 \Rightarrow a = 1.05\text{ m s}^{-2}$	A1	1.045553…
			$a = 0 \Rightarrow v_{steady} = 14.2\text{ m s}^{-1}$	A1 **[5]**	14.194585…
4	(ii)		WD by engine = 23000(10.1)	B1	232300
			WD against resistance = 800(109)	B1	87200
			Change in PE = $1200g(109\sin 4)$	B1	89416.638…
			Change in KE = $\pm\frac{1}{2}(1200)(v^2 - 8^2)$	B1	Use of correct formula for KE to get a change in KE
				M1	Use of conservation of energy, all terms present and no extra terms
			$23000(10.1) = \frac{1}{2}(1200)(v^2 - 8^2) + 800(109) + 1200g(109\sin 4)$	A1	
			$v = 12.5\text{ m s}^{-1}$	A1 **[7]**	12.522204…

Question			Answer	Marks	Guidance
5	(i)		$\sin\theta = \frac{4}{5}$ or $\cos\theta = \frac{3}{5}$ or $\tan\theta = \frac{4}{3}$ or $\theta = 53.1°$	B1	θ is the angle above the horizontal, may be awarded in (ii) or (iii) unless different value already used in (i)
			$R\sin\theta = 0.4g$	M1	Resolving vertically with mass substituted, angle need not be substituted for M1; allow use of their angle including 45°
			$R = \frac{1}{2}g$	A1	
				[3]	
5	(ii)		$r = 3$	B1	May be awarded in (iii) unless different value already used in (ii)
			$R\cos\theta = 0.4(3)\omega^2$ or $\frac{0.4v^2}{3}$	*M1	N2L with cv(r)
			[$\tan\theta = \frac{4}{3} \Rightarrow \omega^2 = \frac{g}{4}$ $\tan\theta = \frac{4}{3} \Rightarrow \omega^2 = \frac{g}{4}$ or $v^2 = \frac{9g}{4}$]	dep*M1	Substituting for R and θ
			Time $= \frac{2\pi}{\sqrt{\frac{g}{4}}} \left(\text{or } \frac{2\pi(3)}{\sqrt{\frac{9g}{4}}}\right) = \frac{4\pi}{\sqrt{g}}$	A1	Use of $\frac{2\pi}{\omega}$ or $\frac{2\pi r}{v}$ to obtain **AG** Correctly shown
				[4]	
5	(iii)		$\sin\alpha = \frac{1}{\sqrt{10}}$ or $\cos\alpha = \frac{3}{\sqrt{10}}$ or $\tan\alpha = 1/3$ or $\alpha = 18.4°$	B1	α is the angle below the horizontal
			$R\sin\theta = T\sin\alpha + 0.4g$	M1	Resolving vertically, 3 terms, use of different angles
			$\frac{4}{5}R = \sqrt{40}\frac{1}{\sqrt{10}} + 0.4g$	A1	$R = 7.4$
				M1	N2L horizontally, 3 terms, use of different angles, a = v^2/r or $r\omega^2$
			$R\cos\theta + T\cos\alpha = \frac{0.4v^2}{3}$ or $0.4(3)\omega^2$	A1	cv(r), do not need a value of R here
			$v = 8.85\,\text{m s}^{-1}$	A1	8.8487287… note that $\omega = 2.94957...$
				[6]	

Question			Answer	Marks	Guidance
6	(i)		CoM of semi-circular lamina of radius r is $\frac{4r}{3\pi}$	B1	oe, may be unsimplified
				M1	Table of values idea
			$2r(10)\left(\frac{10}{2}\right)+\frac{1}{2}\pi r^2\left(10+\frac{4r}{3\pi}\right)=\ldots$	A1	
			$\ldots=\left(2r(10)+\frac{1}{2}\pi r^2\right)\bar{x}$	A1	
			$\bar{x}=\frac{4r^2+30\pi r+600}{3(40+\pi r)}$	A1	**AG** Correctly shown, www.
				[5]	
6	(ii)		$\tan\theta=\frac{r}{10-\bar{x}}$	M1	Using tan to find a relevant angle; accept reciprocal
			$r\left(\frac{120+3\pi r}{1200+30\pi r-600-30\pi r-4r^2}\right)=\frac{1}{4}$	A1	oe
			$(3\pi+1)r^2+120r-150=0$	A1	**AG** Correctly shown
				M1	Attempt to solve quadratic in r
			$r=1.14$ only	A1	1.1375787…
				[5]	

Question			Answer	Marks	Guidance
7	(i)		$4(1)+2(-3)=-4v_A+2v_B$	*M1 A1	Attempt at use of conservation of linear momentum
			$-v_A-v_B=-e(1-(-3))$	*M1 A1	Attempt at use of restitution equation, must be correct way round Must be consistent with direction used for CoLM
				dep*M1	Solve simultaneous equations to find v_A or v_B
			$v_A=\frac{1}{3}(1+4e)$ and $v_B=\frac{1}{3}(8e-1)$	A1	AEF, may be unsimplified.
			$v_B>0 \Rightarrow e>\frac{1}{8}$	A1 [7]	**AG** Correctly shown from correct v_B
7	(ii)		KE before = $\frac{1}{2}(4)(1)^2+\frac{1}{2}(2)(3)^2$	B1	11 J or $\frac{1}{2}(4)(1)^2-\frac{1}{2}(4)\left(\frac{1+4e}{3}\right)^2$
			KE after = $\frac{1}{2}(4)\frac{(1+4e)^2}{9}+\frac{1}{2}(2)\frac{(8e-1)^2}{9}$	B1ft	Allow in terms of 1 variable eg v_A or v_B or e; $\frac{1}{2}(2)(3)^2-\frac{1}{2}(2)\left(\frac{8e-1}{3}\right)^2$
			$11-\frac{2}{9}(1+4e)^2-\frac{1}{9}(8e-1)^2=\frac{5}{2}$	M1	Use KE before – KE after = ±2.5 to get equation in 1 unknown
			$e=\frac{7}{8}$	A1 [4]	**AG** Correctly shown
7	(iii)		$2(-2)+3(3)=2w_B+3w_C$	M1	Attempt at use of conservation of linear momentum, cv(-2)
			$w_B-w_C=5f$	M1	Attempt at use of restitution equation, must be correct way round, cv(-2)
			Use $w_B>\frac{3}{2}$ or $w_C>0$	M1	Dependent on correct NEL and momentum equations with cv, correct cv (3/2) leading to $f>a$ where $0<a\le 1$; allow use of equality to get an inequality for f; $w_B=1+3f$ and $w_C=1-2f$
			$f>\frac{1}{6}, f<\frac{1}{2}$	A1A1	
				[5]	